Beveridgean Unemployment Gap: Online Appendices

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Appendix A. Proofs

We provide proofs that are omitted in the main text.

A.1. Proof that the DMP model's Beveridge curve is strictly convex

In the DMP model, the Beveridge curve $u \mapsto v(u)$ is given by (11):

$$v(u) = \left(\frac{\lambda}{\omega} \cdot \frac{1-u}{u^{\eta}}\right)^{1/(1-\eta)}.$$

Since $(1-u)/u^{\eta} = u^{-\eta} - u^{1-\eta}$, the derivative of the Beveridge curve is

$$v'(u) = \frac{v(u)^{\eta}}{1-\eta} \cdot \frac{\lambda}{\omega} \cdot \left[-\eta u^{-\eta-1} - (1-\eta)u^{-\eta}\right].$$

Reshuffling terms, we obtain

$$v'(u) = -\frac{\lambda}{\omega} \cdot \left[\frac{v(u)}{u}\right]^{\eta} \cdot \left(1 + \frac{\eta}{1-\eta} \cdot \frac{1}{u}\right). \tag{A1}$$

From (A1) we verify that the Beveridge curve is strictly decreasing, because $v'(u) < 0$.

From (A1) we also establish that the Beveridge curve is strictly convex, because $v'(u)$ is strictly increasing in u. Indeed, the second factor in (A1) is strictly decreasing in u because $v(u)$ is strictly decreasing in u and $\eta > 0$. The third factor in (A1) is also strictly decreasing in u because $\eta \in (0, 1)$. Since both factors are positive, their product is strictly decreasing in u. Given that $-\lambda/\omega < 0$, $v'(u)$ is actually strictly increasing in u.

A.2. Proof of proposition 4

We prove the proposition using the auxiliary function

$$G(r, \theta) = \eta\theta + \frac{r+\lambda}{q(\theta)}. \tag{A2}$$

Since $\eta > 0$, $\lambda > 0$, and $q(\theta) = \omega\theta^{-\eta}$ with $\omega > 0$, $G(r, \theta)$ is strictly increasing in θ for any $r \geq 0$.

Characterization of the tightnesses θ^* *and* θ^h. Equation (14) shows that the tightness θ^* given by efficiency condition (4) satisfies

$$G(0, \theta^*) = (1 - \eta)\frac{1 - z}{c}.$$

And (16) shows that the tightness θ^h given by the Hosios condition satisfies

$$G(r, \theta^h) = (1 - \eta)\frac{1 - z}{c}.$$

Thus, for any discount rate r,

$$G(0, \theta^*) = G(r, \theta^h). \tag{A3}$$

Zero discount rate. We begin by considering the case $r = 0$. Equation (A3) implies that $G(0, \theta^*) = G(0, \theta^h)$, so $\theta^* = \theta^h$.

Positive discount rate. Next we consider the case $r > 0$. We assess the gap between θ^* and θ^h by linearizing the function $G(r, \theta)$ around $(0, \theta^*)$. Up to a second-order term, we have

$$G(r, \theta) = G(0, \theta^*) + \frac{\partial G}{\partial r} \cdot r + \frac{\partial G}{\partial \theta} \cdot (\theta - \theta^*), \tag{A4}$$

where the partial derivatives are evaluated at $(0, \theta^*)$. We obtain the partial derivatives from (A2):

$$\frac{\partial G}{\partial r} = \frac{1}{q(\theta^*)}$$
$$\frac{\partial G}{\partial \theta} = \eta + \frac{\lambda}{q(\theta^*)} \cdot \frac{\eta}{\theta^*}.$$

Using these partial derivatives, we evaluate (A4) at (r, θ^h):

$$G(r, \theta^h) = G(0, \theta^*) + \frac{r}{q(\theta^*)} + \eta \left[\theta^* + \frac{\lambda}{q(\theta^*)}\right] \frac{\theta^h - \theta^*}{\theta^*}.$$

Given that $G(r, \theta^h) = G(0, \theta^*)$, we find the relative difference between θ^* and θ^h:

$$\frac{\theta^* - \theta^h}{\theta^*} = \frac{r}{\eta \cdot (f + \lambda)}, \tag{A5}$$

where $f = \theta^* q(\theta^*)$ is the job-finding rate at θ^*.

Shimer calibration. Last, we quantify the relative difference between θ^* and θ^h using the calibration provided by Shimer (2005, table 2): $\eta = 0.72$, $r = 0.012$ per quarter, $\lambda = 0.1$ per quarter, and $f = 1.35$ per quarter. Plugging these numbers into (A5), we find that

$$\frac{\theta^* - \theta^h}{\theta^*} = \frac{0.012}{0.72 \times (1.35 + 0.1)} = 1.1\%.$$

Appendix B. Beveridge curve in the DMP model

In section 4 we argue that in the DMP model the labor market is never far from its Beveridge curve. That is, the actual unemployment rate (given by differential equation (8)) is never far from the Beveridgean unemployment rate (the critical point of (8), given by equation (9)). Here we illustrate the argument using US data for 1951–2019. First, we compute the job-finding rate $f(t)$ and job-separation rate $\lambda(t)$ that through (8) produce the US unemployment rate. Then we compute the corresponding Beveridgean unemployment rate from (9):

$$u^b(t) = \frac{\lambda(t)}{\lambda(t) + f(t)}. \tag{A6}$$

We confirm that the actual unemployment rate closely tracks the Beveridgean unemployment rate.

B.1. Job-finding rate

To compute the job-finding rate, we follow Shimer (2012, pp. 130–133). We first construct the monthly job-finding probability:

$$F(t) = 1 - \frac{u(t+1) - u^s(t+1)}{u(t)}, \tag{A7}$$

where $u(t)$ is the number of unemployed persons in month t, and $u^s(t)$ is the number of persons who have been unemployed for less than 5 weeks in month t (Bureau of Labor Statistics 2020c,d). Assuming that unemployed workers find a job according to a Poisson process with monthly arrival rate $f(t)$, we infer the job-finding rate from the job-finding probability:

$$f(t) = -\ln(1 - F(t)). \tag{A8}$$

Multiplying this monthly rate by 3, we obtain the quarterly job-finding rate in the United States (figure A1). Over 1951–2019, the job-finding rate averages 1.67 per quarter, or 0.558 per month.

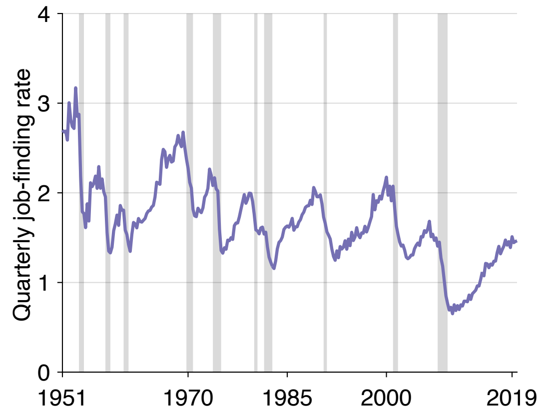


FIGURE A1. Job-finding rate in the United States, 1951–2019

The job-finding rate is constructed from equations (A7) and (A8), as in Shimer (2012). The shaded areas are NBER-dated recessions.

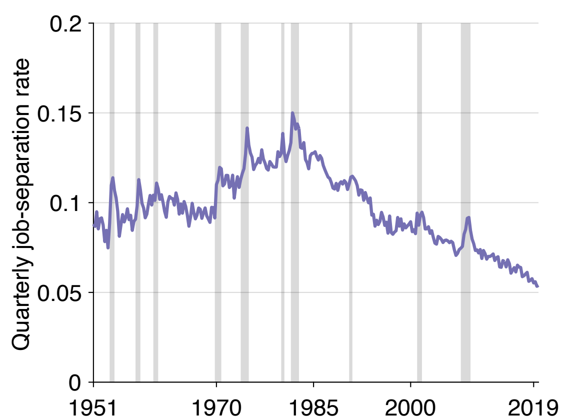


FIGURE A2. Job-separation rate in the United States, 1951–2019

The job-separation rate is constructed from (A9), as in Shimer (2012). The shaded areas are NBER-dated recessions.

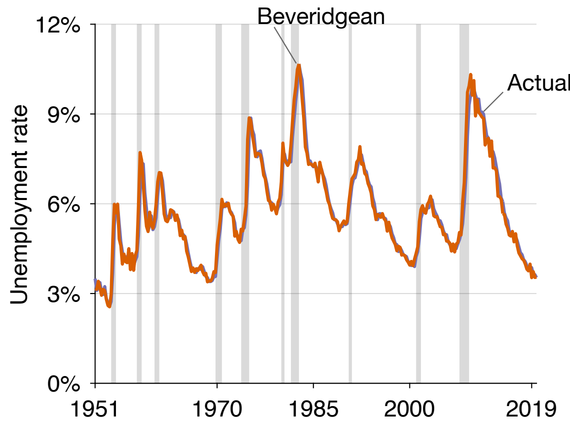


FIGURE A3. Beveridgean unemployment rate in the United States, 1951–2019

The Beveridgean unemployment rate is the unemployment rate on the Beveridge curve of the DMP model. It is constructed using equation (A6), the job-finding rate from figure A1, and the job-separation rate from figure A2. The actual unemployment rate comes from figure 1A; it is displayed as a benchmark. The shaded areas are NBER-dated recessions.

B.2. Job-separation rate

To compute the job-separation rate, we continue to follow Shimer (2012, pp. 130–133). The monthly job-separation rate $\lambda(t)$ is implicitly defined by

$$u(t+1) = \left\{1 - e^{-[f(t)+\lambda(t)]}\right\} \frac{\lambda(t)}{f(t)+\lambda(t)} h(t) + e^{-[f(t)+\lambda(t)]} u(t), \tag{A9}$$

where $f(t)$ is the monthly job-finding rate (given by (A8)), and $h(t)$ and $u(t)$ are the numbers of persons in the labor force and in unemployment (Bureau of Labor Statistics 2020a,d). Each month t, we solve (A9) to compute $\lambda(t)$. Multiplying this monthly rate by 3, we obtain the quarterly job-separation rate in the United States (figure A2). Over 1951–2019, the job-separation rate averages 0.097 per quarter, or 0.032 per month.

B.3. Beveridgean unemployment rate

Finally, we construct the Beveridgean unemployment rate using equation (A6), the job-finding rate from figure A1, and the job-separation rate from figure A2. The Beveridgean unemployment rate is indistinguishable from the actual unemployment rate (figure A3). While the maximum absolute distance between the two series is 1.5 percentage points, the average absolute distance is only 0.2 percentage point.

Appendix C. Endogenous Beveridge elasticity in the DMP model

When we derive the sufficient-statistic formula for the efficient unemployment rate (formula (5)), we assume that the sufficient statistics do not depend on the unemployment and vacancy rates (assumption 3). In the DMP model, however, the Beveridge elasticity depends on the unemployment rate (equation (12)). But in section 4 we argue that the formula should remain accurate because the dependence is weak. Here we confirm this assertion. We calibrate the parameters of the DMP model from US data, 1951–2019. We then compute the efficient unemployment rate in the calibrated DMP model, accounting for the endogeneity of the Beveridge elasticity. We find that the computed efficient unemployment rate is almost identical to the efficient unemployment rate given by formula (5).

C.1. Efficient unemployment rate with endogenous Beveridge elasticity

In the DMP model, when the endogeneity of the Beveridge elasticity is accounted for, formula (14) gives the efficient tightness θ^*:

$$\eta\theta^* + \frac{\lambda}{\omega}(\theta^*)^{\eta} = (1-\eta)\frac{1-z}{c}. \tag{A10}$$

Through the Beveridge curve (9), the efficient tightness θ^* and parameters of the model determine the efficient unemployment rate u^*:

$$u^* = \frac{(\lambda/\omega)}{(\lambda/\omega) + (\theta^*)^{1-\eta}}. \tag{A11}$$

C.2. Application to the United States

Toward applying formulas (A10) and (A11), we calibrate the parameters of the DMP model from US data, 1951–2019.

Social value of nonwork and recruiting cost. As in section 5, we set the social value of nonwork to $z = 0.26$ and the recruiting cost to $c = 0.92$.

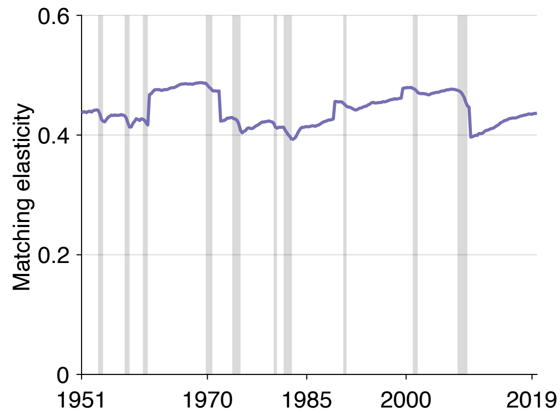


FIGURE A4. Matching elasticity in the United States, 1951–2019

The matching elasticity is computed using equation (A12), the Beveridge elasticity from figure 6, and the unemployment rate from figure 1A. The shaded areas are NBER-dated recessions.

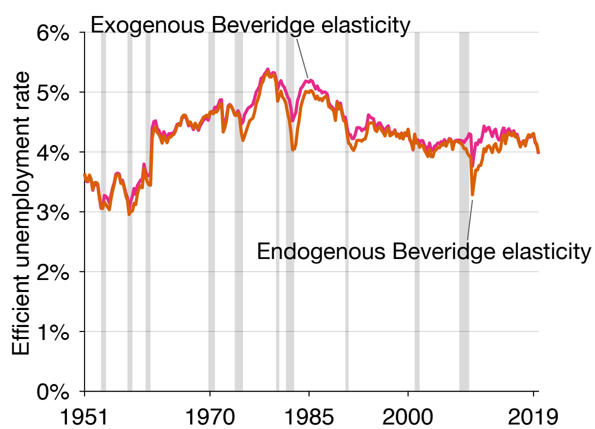


FIGURE A5. US efficient unemployment rate with endogenous Beveridge elasticity

The efficient unemployment rate with endogenous Beveridge elasticity accounts for the endogeneity of the Beveridge elasticity that appears in the DMP model. It is constructed by solving equations (A10) and (A11). The efficient unemployment rate with exogenous Beveridge elasticity comes from figure 7B; it is displayed as a benchmark. The shaded areas are NBER-dated recessions.

Matching elasticity. By inverting equation (12), we express the matching elasticity η as a function of the Beveridge elasticity ϵ:

$$\eta = \frac{1}{1+\epsilon}\left(\epsilon - \frac{u}{1-u}\right). \tag{A12}$$

We then compute the matching elasticity from the Beveridge elasticity in figure 6 and the unemployment rate in figure 1A. Between 1951 and 2019, the matching elasticity averages 0.44, and it always remains between 0.39 and 0.49 (figure A4).

Separation-efficacy ratio. As we assume that unemployment is always on the Beveridge curve, labor flows are balanced, so $\lambda(1-u) = f(\theta)u = \omega\theta^{1-\eta}u$. Therefore, the ratio of the job-separation rate λ to the matching efficacy ω satisfies

$$\frac{\lambda}{\omega} = \frac{u}{1-u} \cdot \theta^{1-\eta}.$$

We compute the ratio λ/ω from this relation, the unemployment rate in figure 1A, the tightness in figure 7A, and the matching elasticity in figure A4.

Efficient unemployment rate. Plugging the parameter values into formulas (A10) and (A11), we compute the efficient unemployment rate in the DMP model (figure A5). This efficient unemployment rate accounts for the endogeneity of the Beveridge elasticity that arises in the DMP model. Yet it closely tracks the baseline efficient unemployment rate, which takes the Beveridge elasticity as exogenous. The maximum absolute distance between the two series is 0.5 percentage point, and the average absolute distance is only 0.1 percentage point.

Appendix D. Hosios condition in the DMP model

Proposition 4 establishes that in the DMP model, the efficient tightness given by the Hosios condition is almost identical to the efficient tightness arising from our Beveridgean approach. Here we simulate a DMP model calibrated to US data, 1951–2019, and we show that the efficient unemployment rates given by the Hosiosian and Beveridgean approaches also are almost identical.

D.1. Efficient unemployment rate given by the Hosios condition

In the DMP model, the efficient tightness θ^h given by the Hosios condition satisfies (16):

$$\eta\theta^h + \frac{\lambda + r}{\omega}(\theta^h)^\eta = (1-\eta)\frac{1-z}{c}. \tag{A13}$$

Then, the efficient unemployment rate u^h given by the Hosios condition solves differential equation (8), where the job-finding rate is $f = f(\theta^h)$. Accordingly, we compute $u^h(t)$ recursively. We initialize $u^h(1) = u(1)$. We then iterate (10):

$$u^h(t+1) = u^b(\theta^h) + [u^h(t) - u^b(\theta^h)]e^{-[\lambda + f(\theta^h)]}, \tag{A14}$$

where $f(\theta^h) = \omega(\theta^h)^{1-\eta}$ and $u^b(\theta^h) = \lambda/[\lambda + f(\theta^h)]$.

D.2. Application to the United States

To apply formulas (A13) and (A14), we calibrate the parameters of the DMP model from US data, 1951–2019.

Social value of nonwork, recruiting cost, and matching elasticity. As in appendix C, we set the social value of nonwork to $z = 0.26$ and the recruiting cost to $c = 0.92$, and we take the matching elasticity η from figure A4.

Job-separation rate. We take the quarterly job-separation rate λ from figure A2.

Discount rate. As in Shimer (2005, table 2), we set the quarterly discount rate to $r = 0.012$, which corresponds to an annual discount rate of 5%.

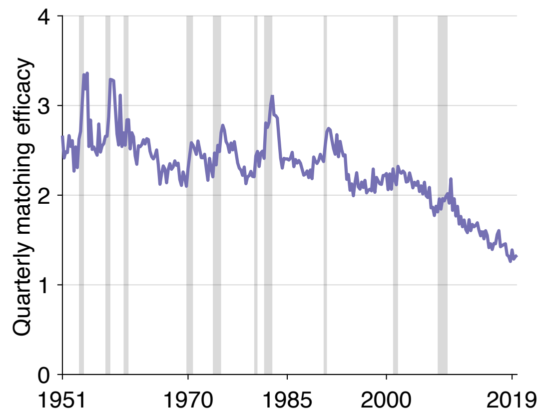


FIGURE A6. Matching efficacy in the United States, 1951–2019

The matching efficacy is constructed using equation (A15), the labor-market tightness from figure 7A, the quarterly job-finding rate from figure A1, and the matching elasticity from figure A4. The shaded areas are NBER-dated recessions.

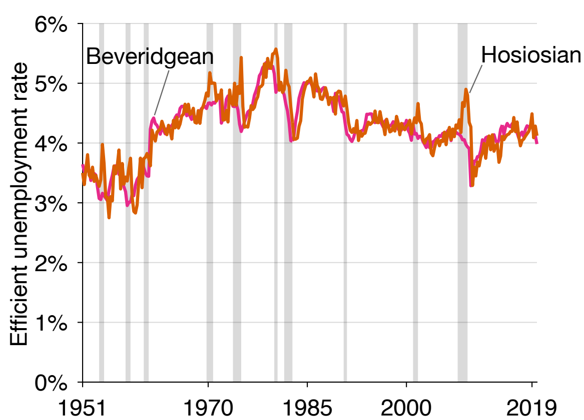


FIGURE A7. Hosiosian efficient unemployment rate in the United States, 1951–2019

The Hosiosian efficient unemployment rate accounts for the dynamics of unemployment in the DMP model. It is constructed by solving equation (A13) and iterating equation (A14). The Beveridgean efficient unemployment rate comes from figure A5; it is displayed as a benchmark. The shaded areas are NBER-dated recessions.

Matching efficacy. With the matching function (6), the job-finding rate is $f = \omega\theta^{1-\eta}$, so the matching efficacy satisfies

$$\omega = \frac{f}{\theta^{1-\eta}}. \tag{A15}$$

We compute the quarterly matching efficacy from (A15), the tightness in figure 7A, the quarterly job-finding rate in figure A1, and the matching elasticity in figure A4; the result is displayed in figure A6.

Efficient unemployment rate. Plugging the parameter values into formulas (A13) and (A14), we compute the efficient unemployment rate given by the Hosios condition. We find that this Hosiosian efficient unemployment rate is close to the Beveridgean efficient unemployment rate computed in appendix C (figure A7). While the maximum absolute distance between the two series is 1.1 percentage point, the average absolute distance is only 0.2 percentage point. Moreover, the difference between the two series is not due to differences in the efficient tightnesses; rather, it is due to the conversion of tightness into unemployment.

Appendix E. Fluctuating social value of nonwork in the DMP model

When we compute the US unemployment gap (figure 7), we keep the social value of nonwork constant. This choice is justified by the work of Chodorow-Reich and Karabarbounis (2016), who find that the social value of nonwork is acyclical. In some versions of the DMP model, however, the productivities of unemployed and employed workers do not move in tandem over the business cycle, which generates fluctuations in the social value of nonwork. Here we show that such fluctuations have virtually no effect on the efficient unemployment rate.

E.1. Efficient unemployment rate with fluctuating social value of nonwork

In the DMP model, the social value of nonwork is constant when the productivity of unemployed workers is proportional to the productivity of labor (equation (13)). While such proportionality necessarily holds in the long run, it could fail in the short run. In that case, short-run fluctuations in labor productivity are nonneutral: they create fluctuations in the social value of nonwork and in the efficient unemployment rate (Shimer 2005).

To introduce fluctuations of the social value of nonwork in the DMP model, we assume that the productivity of unemployed workers is proportional to the trend of labor productivity, $\bar{p}$, instead of actual labor productivity, p. Under this specification, the welfare function (2) becomes

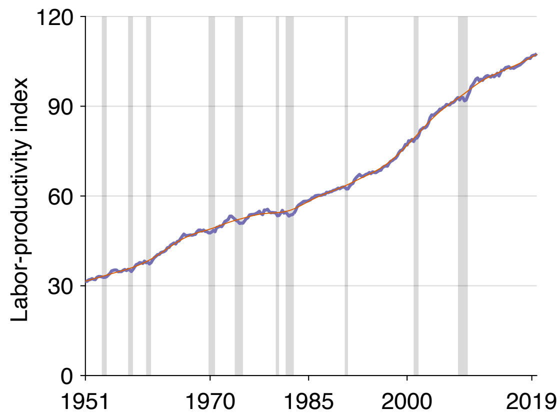
$$\mathcal{W}(n, u, v) = (pn + \bar{p}zu - pcv)\, L. \tag{A16}$$

The social value of nonwork becomes

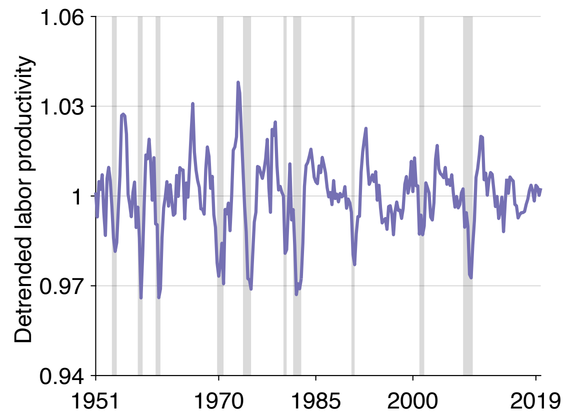
$$\zeta = \frac{z}{\hat{p}},$$

where $\hat{p} = p/\bar{p}$ is detrended labor productivity. Formula (5) therefore becomes

$$u^{*} = \left[\frac{c\epsilon}{1 - (z/\hat{p})} \cdot \frac{v}{u^{-\epsilon}}\right]^{1/(1+\epsilon)}. \tag{A17}$$



A. Labor productivity and trend



B. Detrended labor productivity

FIGURE A8. Labor productivity in the United States, 1951–2019

A: Labor productivity is the index of real output per worker constructed by the Bureau of Labor Statistics (2020b). The trend of productivity is produced by a HP filter with smoothing parameter 1600. B: Detrended labor productivity is the labor productivity from panel A divided by its trend. The shaded areas are NBER-dated recessions.

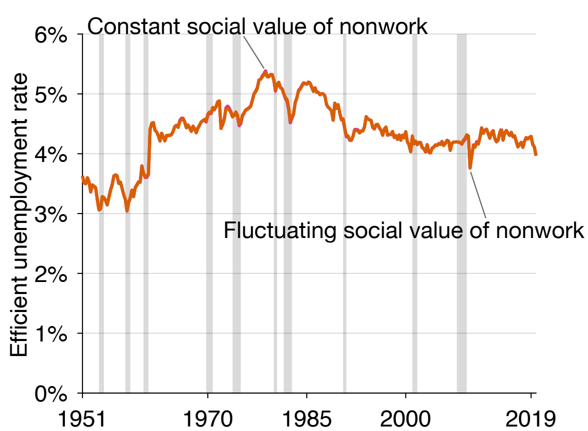


FIGURE A9. US efficient unemployment rate with fluctuating social value of nonwork

The efficient unemployment rate with fluctuating social value of nonwork incorporates the fluctuations of the social value of nonwork that appear in the DMP model when social welfare is given by (A16). It is constructed using equation (A17) and the detrended productivity from figure A8. The efficient unemployment rate with constant social value of nonwork comes from figure 7B; it is displayed as a benchmark. The shaded areas are NBER-dated recessions.

E.2. Application to the United States

To apply formula (A17), we measure detrended labor productivity and the other statistics in the United States.

Detrended labor productivity. We measure labor productivity (p) in the United States from the real output per worker constructed by the Bureau of Labor Statistics (2020b). We compute the trend of productivity ($\bar{p}$) using a HP filter; since the productivity series has quarterly frequency, we set the filter's smoothing parameter to 1600 (Ravn and Uhlig 2002). We compute detrended labor productivity as $\hat{p} = p/\bar{p}$ (figure A8).

Other statistics. As in section 5, we set the average social value of nonwork to $z = 0.26$ and the recruiting cost to $c = 0.92$, and we take the Beveridge elasticity ϵ from figure 6. We also take the vacancy rate v and unemployment rate u from figure 1.

Efficient unemployment rate. Finally, using formula (A17), we compute the efficient unemployment rate in the United States when the social value of nonwork fluctuates over the business cycle (figure A9). We find that the efficient unemployment rates with and without fluctuations of the social value of nonwork are indistinguishable. The maximum absolute distance between the two series is only 0.03 percentage point.

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