Individual and Bilateral Surpluses from Trade

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Utility function.

$$u\left(c, \frac{m}{p}\right) = \frac{\chi}{1+\chi} c^{\frac{\varepsilon-1}{\varepsilon}} + \frac{1}{1+\chi} \left(\frac{m}{p}\right)^{\frac{\varepsilon-1}{\varepsilon}}$$

Marginal utility of services:

$$\frac{\partial u}{\partial c} = \frac{\chi}{1+\chi} \frac{\varepsilon-1}{\varepsilon} c^{-1/\varepsilon}$$

→ buyer takes away from trade

Marginal utility of money.

$$\frac{\partial u}{\partial m} = \frac{1}{1+\chi} \cdot \frac{\varepsilon-1}{\varepsilon} \left(\frac{m}{p}\right)^{-1/\varepsilon} \frac{1}{P}$$

← aggregate price level

For a transaction at price p^n (price norm), utility is

$$p^n \cdot \frac{\partial u}{\partial m} = \frac{p^n}{P} \cdot \frac{1}{1+\chi} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot \left(\frac{m}{p}\right)^{-1/\varepsilon}$$

All households hold μ units of money, so seller experience utility.

$$p^n \frac{\partial u}{\partial m} = \left(\frac{p^n}{P}\right) \frac{1}{1+\chi} \frac{\varepsilon-1}{\varepsilon} \cdot \left(\frac{\mu}{P}\right)^{-1/\varepsilon}$$

seller takes away from trade.

If there was no trade (required to compute surplus).

- seller: gets 0.

- buyer gets p^n units of money, which provide utility:

$$p^n \frac{\partial u}{\partial m} = \left(\frac{p^n}{p}\right) \frac{1}{1+X} \quad \frac{\varepsilon-1}{\varepsilon} \cdot \left(\frac{N}{p}\right)^{-1/\varepsilon}$$

- Seller enjoys surplus from trade:

$$S = \left(\frac{p^n}{p}\right) \frac{1}{1+X} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot \left(\frac{N}{p}\right)^{-1/\varepsilon}$$

All prices are the same, given by price norm, so $p^n = p$.

$$\boxed{S = \frac{1}{1+X} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot \left(\frac{N}{p}\right)^{-1/\varepsilon}} \qquad \boxed{S > 0}$$

- Buyer enjoys surplus from trade:

$$B = \frac{X}{1+X} \cdot \frac{\varepsilon-1}{\varepsilon} \quad c^{-1/\varepsilon} - \left(\frac{p^n}{p}\right) \frac{1}{1+X} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot \left(\frac{N}{p}\right)^{-1/\varepsilon}$$

$$B = \frac{1}{1+X} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot \left[X c^{-1/\varepsilon} - \left(\frac{p^n}{p}\right)\left(\frac{N}{p}\right)^{-1/\varepsilon} \right]$$

$$B = \frac{1}{1+X} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot \left[X c^{-1/\varepsilon} - \left(\frac{N}{p}\right)^{-1/\varepsilon} \right] \quad \text{(all prices are same)}$$

FOC from household's problem: (used to compute AD curve)

$$X c^{-1/\varepsilon} = [1+\tau(x)] \, (N/p)^{-1/\varepsilon}$$

$$B = \frac{1}{1+x} \cdot \frac{\varepsilon-1}{\varepsilon} \cdot \left[(1+\tau(x)) (w/p)^{-1/\varepsilon} - (w/p)^{-1/\varepsilon} \right]$$

$$\boxed{B = \frac{1}{1+x} \cdot \frac{\varepsilon-1}{\varepsilon} \quad \tau(x) \cdot (w/p)^{-1/\varepsilon}} \qquad \boxed{B > 0}$$

Conclusion:

- seller surplus, $\boxed{S > 0}$
- buyer surplus: $\boxed{B > 0}$
- total surplus from match:

$$\boxed{T = S + B > 0}$$