PROBLEM SET ON SOLOWIAN MODEL OF GROWTH

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Problem 1

Consider the production function $Y = \sqrt{N}\sqrt{K}$, where Y is output, N is labor, and K is capital.

A) Derive the relation between the growth rate of output, g_Y, the growth rate of capital, g_K, and the growth rate of labor, g_N.

B) Assume that N is constant, and suppose we want to achieve output growth equal to 2% per year. What is the required rate of growth of capital?

C) In the situation described in B), what happens to the ratio of capital to output, K/Y, over time?

Problem 2

Consider a Solow model with production function $Y = N^{1-a} \times K^a$, where Y is output, N is labor, K is capital, and $0 < a < 1$ is a parameter. The number of workers, N, is fixed. The saving rate is $s > 0$ and the depreciation rate is $\delta > 0$.

A) Use the production function to compute output per worker y as a function of capital per worker k.

B) Express investment per worker as a function of the saving rate s and capital per worker k. What are the two assumptions required to reach this result?

C) Find an equation relating capital in period t+1, K(t+1), to capital in period t, K(t), and investment in period t, I(t).

D) Using the results to questions B) and C), derive the law of motion for capital per worker. That is, find an equation relating capital per worker in period t+1, k(t+1), to capital per worker in period t, k(t), and parameters of the model.

E) Derive the steady-state level of capital per worker in terms of the saving rate, s, and the depreciation rate, δ, and the production-function parameter, a.

F) Derive the steady-state levels of output per worker and consumption per worker in terms of the saving rate, s, and the depreciation rate, δ, and the production-function parameter, a.

G) Set $a = 0.5$ and $\delta = 5\%$. Compute steady-state output per worker and steady-state consumption per worker for $s = 0\%$; $s = 5\%$; $s = 10\%$; $s = 20\%$; $s = 30\%$; $s=50\%$; $s=70\%$; and $s=100\%$. Explain the intuition behind your results.

H) Given a production-function parameter a and depreciation rate δ, find the value of the saving rate s that maximizes consumption per worker. What is the name of this value?

I) Compute consumption per worker when the saving rate takes the value obtained in question H).

J) How does the value of the saving rate found in H) and the value of consumption found in I) depend on the production-function parameter a and depreciation rate δ? Discuss.

Problem 3

Consider a Solow model with production function $Y = K^{2/3} \times N^{1/3}$. The saving rate is 10% and the depreciation rate is 6%. The number of workers in the economy is $N = 100$.

A) In steady state, at what rates do output, output per worker, consumption, and consumption per worker grow? Why?

B) Draw the equilibrium diagram of this Solow model. Show where the steady state is. Give the equations for all the curves you have placed on the diagram.

C) Solve for capital per worker and output per worker in steady state.

D) Solve for consumption per worker and investment per worker in steady state.

E) Does the economy have more or less capital per worker than at the golden-rule steady state? To achieve the golden-rule steady state, does the saving rate need to increase or decrease?

F) Suppose the change in saving rate described in question E) occurs. Draw the evolution of consumption per worker over time: in the old steady state, after the change in saving rate, and in the golden-rule steady state. Explain.

G) Suppose the change in saving rate described in question E) occurs. Draw the evolution of output per worker over time: in the old steady state, after the change in saving rate, and in the golden-rule steady state. Explain.